NAFEMS Benchmark Challenge

Challenge Problem 2 - The Solution

A building has a floor opening that has been covered by a durbar plate with a yield stress of 275MPa. The owner has been instructed by his insurers that for safety the load carrying capacity of the plate needs to be assessed. The owner has calculated (possibly unrealistically but certainly conservatively) that if 120 people each weighing 100kg squeeze onto the plate then it must be able to cope with 100kN/m². He has found, in the Steel Designers' Manual, that the plate should be able to withstand 103kN/m². This is rather close to the required load and looking in Roark's Formulas for Stress and Strain he finds that the collapse load is more like 211kN/m² which he feels does provide an adequate factor of safety. However, with the huge difference between the two published values he has asked you to provide him with an independent assessment of the load carrying capacity of the plate.

The Challenge

As an experienced engineer you realise that under increasing load the plate will eventually reach first yield after which the stress will redistribute until the final collapse load is reached. You will appreciate that the steel will have some work hardening capability and that if transverse displacements are considered then some membrane action will occur. However, opting for simplicity and realising that ignoring these two strength enhancing phenomena will lead to a degree of conservatism in your assessment, you decide that this is a limit analysis problem in which the flexural strength of the plate governs collapse.

Unless you have specialist limit analysis software you will decide to tackle this as an incremental non-linear plastic problem with a bi-linear stress/strain curve and a von Mises yield criterion.

Please carry out an assessment of the strength of the plate and provide your best estimate of the actual collapse load together with evidence of the verification you have conducted sufficient to convince the owner and his risk averse insurer.

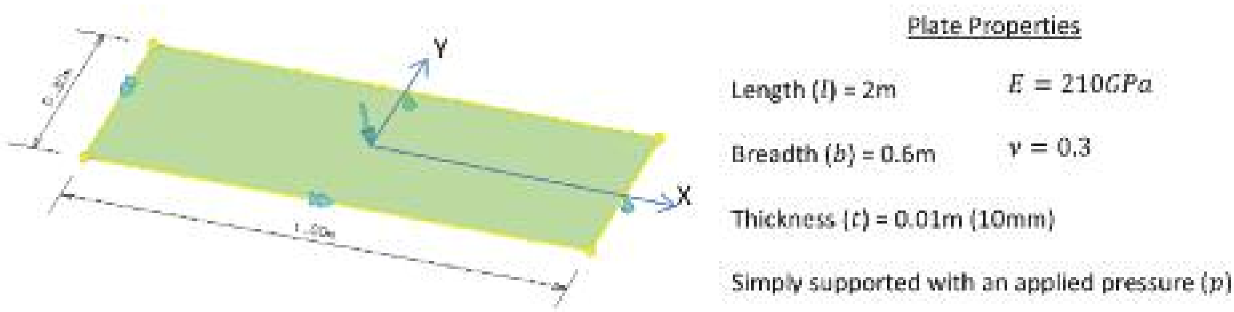


Figure 1: Symmetric quarter model of challenge plate and boundary conditions

Raison d'être for the Challenge

This challenge derives from an observation that the strength of steel plates, such as the one in this challenge, as quoted in the Steel Designers' Manual is significantly lower than that derived from standard reference texts such as Roark. The strength of a steel plate such as the challenge plate can be assessed using limit analysis techniques. Challengers were asked to tackle the problem using nonlinear incremental limit analysis.

Steel Designers' Manual
(6th Edition, p1281)

Ultimate load capacity (kN/m²) for floor plates simply supported on all four edges stressed to 275 N/mm² (Values obtained using Pounder's formula allowing corners to lift)

Thickness on plain mm	Breadth B mm	Length (mm) 600	800	1000	1200	1400	1600	1800	2000
10.0	600	172*	126*	112*	107*	105*	103*	103*	103*
	800		97.0	74.5	65.9	62.1	60.1	59.1	58.5
	1000			62.1	49.5	43.9	41.0	39.4	38.5
	1200				43.1	35.4	31.5	29.3	28.0
	1400					31.7	26.6	23.8	22.1
	1600						24.3	20.7	18.6
	1800							19.2	16.6

Values *without* an asterisk cause deflection greater than b/100 at serviceability, assuming that the only dead load present is due to self-weight.

Roark's Formulas for Stress and Strain
(7th Edition, pp453-454)

11.13 Ultimate Strength

Plates of brittle material fracture when the actual maximum tensile stress reaches the ultimate tensile strength of the material. A flat-plate modulus of rupture, analogous to the modulus of rupture of a beam, may be determined by calculating the (fictitious) maximum stress corresponding to the breaking load, using for this purpose the appropriate formula for elastic stress. This flat-plate modulus of rupture is usually greater than the modulus of rupture determined by testing a beam of rectangular section.

Plates of ductile material fail by excessive plastic deflection, as do beams of similar material. For a number of cases the load required to produce collapse has been determined analytically, and the results for some of the simple loadings are summarized as follows.

3. *Rectangular plate, length a, width b; uniform load, edges supported*

$$W_u = \beta \sigma_y t^2$$

where β depends on the ratio of b to a and has the following values (Ref. 44):

b/a	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
β	5.48	5.50	5.58	5.64	5.89	6.15	6.70	7.68	9.69

In each of the above cases W_u denotes the total load required to collapse the plate, t the thickness of the plate, and σ_y the yield point of the material. Accurate prediction of W_u is hardly to be expected; the theoretical error in some of the formulas may range up to 30%, and few experimental data seem to be available.

Review of Published Strength Values Corrigendum – The Owner Bodged his Calculation!

Sharp readers will have noted that the owner of the building made a mistake using the equation from Roark. He calculated W_u correctly but then assumed that this was the applied pressure or UDL. However, this value is the total load and so the pressure is obtained by dividing the total load by the area of the plate:

$$p = \frac{W_u}{Area} = \frac{7.68 \cdot 275E^6 \cdot 0.01^2}{2 \cdot 0.6} = 176kPa$$

It is interesting to notice that whilst the collapse pressure is dependent on the area of the plate, the collapse load is not.

Plate Theory and Boundary Conditions

Classical plate theory recognises the need for different formulations for 'thick' and 'thin' plates. For 'thick' plates the appropriate theory is Reisner-Mindlin, which accounts for shear deformation, whereas for 'thin' plates the appropriate theory is Kirchhoff which assumes the section to be rigid in terms of shear deformation. The transition between 'thick' and 'thin' plates occurs at a span to depth ratio of about ten and as the challenge problem has a ratio of 0.6/0.01=60 it is clearly in the realm of 'thin' plate or Kirchhoff theory. In theory, at least, the results produced by 'thick' plate theory should converge to those achieved by 'thin' plate theory as the span to depth ratio increases. However, one needs to be aware that with many finite element formulations of 'thick' plates, a nasty phenomenon known as 'shear-locking' may occur to stymy this principal. Fortunately for the challenge problem the span to depth ratio is not sufficiently large that shear-locking will occur and finite elements of either formulation can be used successfully to solve this problem.

The formulation used for a plate element has implications on the nature of the boundary conditions that might be applied. The thick formulation allows torsional moments and their corresponding twisting rotation (about an axis perpendicular to the surface of the plate) to be specified at the boundary whereas the thin formulation does not explicitly control these quantities. As such, for a thick plate, the support conditions may be specified as 'soft' or 'hard' depending on whether the twisting rotations are free or constrained. As the challenge problem is a thin plate problem this distinction is not relevant BUT if the challenge plate is modelled with a thick plate finite element formulation then the simple supports should be interpreted as hard simple supports and the twisting rotation set to zero.

As with any problem that exhibits symmetry it is worth taking advantage of this property in order to reduce the size of the finite element model and therefore the time to compute a solution.

Elastic Solution

A good starting point for understanding the challenge problem is to look at the elastic solution. This problem was analysed using two popular commercial finite element tools (CS1 and CS2). CS1 was used to generate the elastic solutions and the results for a load of 100kPa are shown in figure 2.

From the elastic analysis we have sufficient results to make the following statements:

1. The peak von Mises stress is 232MPa for a load of 100kPa so that first yield is reached at 275/232=1.185 times the applied pressure which is 118.5kPa.

2. Even if no redistribution occurs across the face of the plate, the applied pressure to cause plastic collapse is 1.5x118.5=178kPa – where the 1.5 factor comes from the increase in load required to develop yield from first yield at the surface to full yield through the section.

It is curious that this value is a little greater than the Roark value of 176kPa. So the Roark value is clearly too conservative especially, since redistribution across the plate does occur!

The Steel Designers' Manual (SDM) use Pounder's equations which are reproduced from his paper [2] below:

Max Skin Stress - Pounder equation 19

$$\frac{3}{4}\frac{kpb^2}{t^2}\left\{1+\frac{14}{75}(1-k)+\frac{20}{57}(1-k)^2\right\}$$

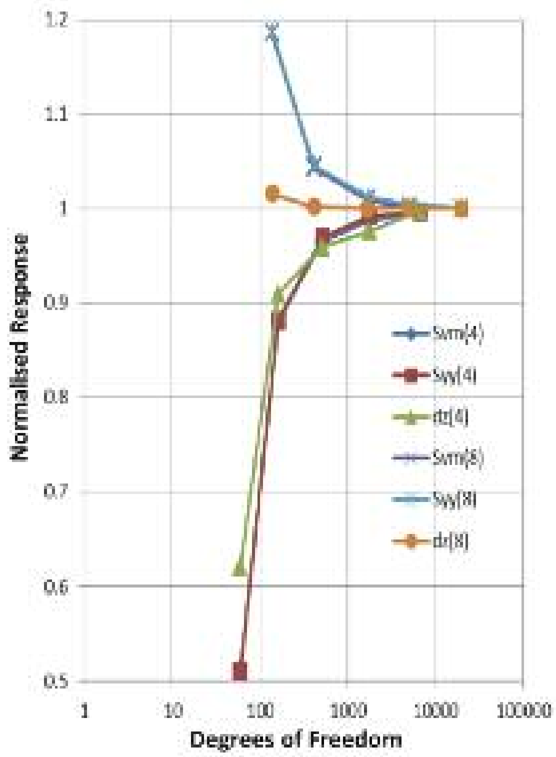
Maximum Deflection - Pounder equation 19(a)

$$\frac{\nu^2-1}{\nu^2}\cdot\frac{5kpb^4}{32Et^3}\cdot\left\{1+\frac{37}{175}(1-k)+\frac{79}{201}(1-k)^2\right\}$$

where:

$$k=\frac{l^4}{l^4+b^4}$$

Pounder's equations are based on an elastic analysis and for a pressure of 100kPa applied to the challenge plate give a maximum direct stress of 268MPa (which is pleasingly close to the finite element value of 262MPa) and a maximum displacement of 8.72mm (again close to the finite element value of 8.5mm). Comparing the maximum stress with the yield stress gives 275/268=1.03 which corresponds to an applied pressure of 103kPa and is identical to the value reported in the SDM. The approach used by the SDM then is one which limits the maximum principal stress to the yield stress for the material. Thus whilst the SDM quotes these loads as 'Ultimate Load Capacity' the method allows no plastic redistribution and whilst



The maximum displacement and stresses occur at the centre of the plate and this figure shows the convergence of these quantities with mesh refinement for both four-noded and eight-noded elements. The values have been normalised by the values achieved with the most refined eight-noded mesh.

Svm = 232MPa; Syy = 262MPa; dz = 8.5mm. The normalised peak von Mises stress and peak Syy stress are virtually identical and so the curves lie on top of each other in the figure.

It is interesting to note that the direction in which convergence occurs is dependent on the element type with results converging from above the true solution for the eight-noded element and below the true solution for the four-noded element.

Figure 2: Convergence of stress and displacement for an elastic analysis of the challenge plate

b/a	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
β (Roark)	5.48	5.50	5.58	5.64	5.89	6.15	6.70	7.68	9.69
β (EFE)	6.26	6.28	6.40	6.51	6.90	7.42	8.38	10.08	13.67
Difference	14%								41%

Table 1: Roark's values for β appended with those produced by specialist limit analysis software

appropriate for brittle materials, is not relevant for the sort of ductile plates being considered – see figure 5 for practical evidence of ductility. In terms of deflection, then the deflection at 'failure' is 1.03x8.72=9mm. It is interesting to note that in the SDM, and based on a breadth/100=6mm deflection limit, the displacement is greater than this value so the asterisk indicating that SLS is not a concern is incorrect!

Pounder's paper does not appear to deal with the uplift phenomenon and this is not surprising since corner uplift is a non-linear phenomenon that is not simply dealt with in the form of linear analysis considered in Pounder's paper. To consider corner uplift one can perform an incremental finite element analysis progressively releasing any boundary nodes where the reactions are tensile. Such an analysis has been performed with linear elastic material properties and although not reported here, the change in the maximum stress and displacement is observed to be minimal.

Target Solution Solution

The idea of limit analysis is to find the collapse load of the structure based on plasticity arguments and a perfectly-plastic material model. For plates such as the challenge plate it is the flexural failure mode that dominates. A benchmark target solution has been created using a specialist limit analysis software which allows the collapse solution and load to be found directly without recourse to incremental application of the pressure load. The benchmark collapse load comes out at about 231kPa.

The pressure to produce first yield has already been calculated as 118.5kPa which leads (without stress redistribution across the surface of the plate) to a conservative prediction of collapse at 178kPa. The value achieved by our benchmark solution indicates that significant stress redistribution does occur across the surface of the plate.

It is interesting to compare the collapse pressures produced by a specialist limit analysis software tool using the equilibrium finite element method (EFE) with those tabulated in Roark for different plate aspect ratios (Table 1).

It can be seen that by using numerical analysis more capacity can be achieved from one's structure – the results from specialist limit analysis software are greater than those produced by Roark. In the text, Roark admits that the results are not expected to be accurate and that they could be up to 30% in error. In practice whilst Roark's values are conservative, for the plate aspect ratios considered, they can be greater than 40% in error!

Incremental Limit Analysis

Collapse pressures for the challenge plate were generated using fairly coarse meshes in the two commercial finite element systems. A shell element with nine integration points through the thickness was utilised. This integration scheme was chosen after testing the element performance in a simple representative benchmark, see Appendix 1 for details.

	CS1		CS2	
Mesh	Reduced	Full	Reduced	Full
1x3	387	385	1000 +	386
3x10	235	235	1000 +	241
6x20	225	224	435	234

Table 3: Collapse pressures (kPa) for two commercial FE systems (challenge problem)

The first point to note with regard to the performance of the commercial FE codes on the challenge problem is that convergence is from above the true value. This is important since, at least for the challenge problem, coarse meshes produce unsafe predictions of the collapse loads. It appears also to be the case that reduced integration schemes may have a detrimental effect on the prediction of the collapse pressure. This is particularly significant for CS2 which would appear to show that the entire pressure can be taken for the coarser meshes with the reduced integration element. Despite the above concerns, it is pleasing to see, at least for the fully integrated elements, that the collapse load is predicted fairly reasonably (at least in an engineering sense) by both codes. The convergence of the collapse pressure for the challenge plate is shown in figure 3. In discussion with the developers of CS2 it was discovered that the reason for the poor performance of the reduced integration element was the inclusion of an artificial bending stiffness to eliminate the potential for exciting bending hour glass modes of deflection.

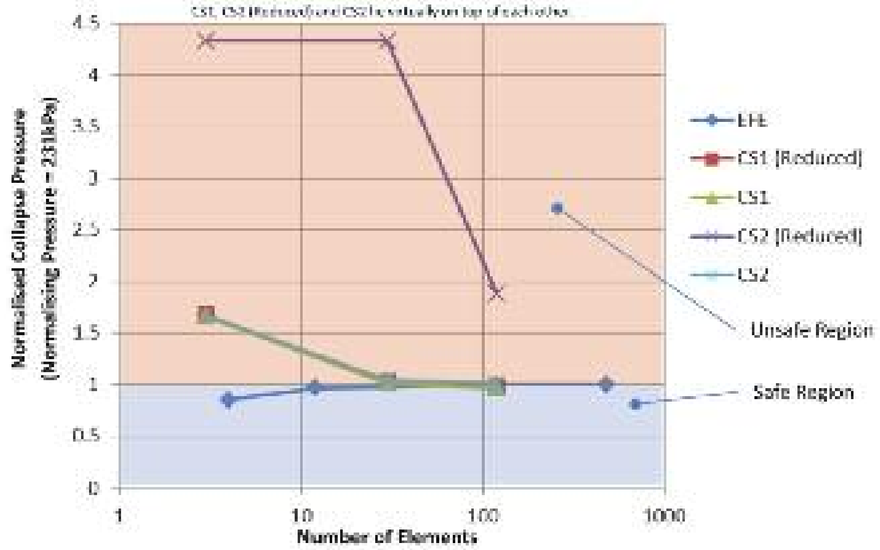


Figure 3: Convergence of the collapse pressure for the challenge plate

Discussion

The results published in the SDM are based on the elastic equations of Pounder which. The SDM uses Pounder's equations to predict the maximum principal stress in the plate and then factors this with the yield stress to provide what it calls the 'Ultimate Load Capacity'. This phrase is generally understood to mean the load at which failure will occur and in the context of steel plates one would generally interpret this as the load at which failure by collapse occurs. However, it is clear, from this study, that the meaning implied in the SDM is the load at which first yield occurs. This might of course mean actual failure if the plate were made of a brittle material (Pounder's work did deal with cast steel plates) but as used by the SDM in the context of Durbar plates, which offer considerable ductility this is rather misleading.

In contrast to the SDM, Roark makes clear that it is talking about the plastic collapse of plates so we understand the loads it provides to be collapse loads. The results are produced in tabular format for a practical range of plate aspect ratios. The result for the challenge plate aspect ratio is interesting since it is less than 1.5 times the load to produce first yield. It is known, even without allowing for any plastic redistribution across the plate, that the redistribution through the thickness will give a collapse pressure 1.5 times that required to produce first yield and so, as in general redistribution across the plate can and does occur (see appendix), it is clear that this result is going to be rather conservative.

Published collapse loads for plates aimed at helping the practicing engineer are seen to be significantly different (although conservative) than the result produced through limit analysis. This means that engineers using these texts will design structures with more material than is really necessary and/or they will underestimate the residual strength when reassigning the duty of a structural plate element potentially unnecessarily requiring it's strengthening or replacement. Neither situation is satisfactory. Commercially available finite element software tools are able to simulate limit analysis solutions with reasonable accuracy BUT the results they produce are strongly dependent on the engineer being skilled in non-linear FEA and having the time and interest in ensuring appropriate simulation governance or verification.

Limit analysis is conservative since it ignores strength enhancing phenomenon such as strain hardening and membrane actions. Whilst large permanent deformations might, in some applications, be considered unacceptable, there are plenty of examples of plates, such as the challenge plate, where large permanent deformations do not affect the serviceability of the plate - see figure 4 for example.

Figure 4: Permanent and large deflections of a plate

The challenge, as presented, was for engineers to 'convince the owner of the building and his risk averse insurer' that the plate, as fitted by the owner, was fit for duty. Given that the collapse pressure is 231kPa then this is some 2.3 times the maximum load the plate is ever likely to see.

The owner of the building (Ferdinand Frugal of Frugal Castle) was rather peeved at the outcome of this study in that he realises he could have got away with using a thinner (8mm instead of 10mm) plate:

The difference in the cost of the plate is some £64 which Ferdinand feels would have been better spent on a case of cheap Whisky for his guests!

Product	Description	UOS	Qty	Price	Value	
25016210	Mild Steel Durbar 2000 x 1000 x 8mm Grade S235	EACH	1	£121.18	£121.18	Delete
250163110	Mild Steel Durbar 2000 x 1000 x 10mm Grade S235	EACH	1	£173.94	£173.94	Delete

Update

Standard Delivery charges apply to the UK Mainland only. For delivery outside of the UK Mainland please contact your local branch.

Goods Value	£295.12
VAT Value	£59.02
Invoice Total	£354.14

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Figure 5: Cost of Durbar plate for the challenge problem